Recasting The Dividend Discount Model As A Return Model

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In this white paper we will recast the Dividend Discount Model as a Return Model. For that purpose we will use the following hypothetical problem...

Our Hypothetical Problem

Assume that we are tasked with calculating the enterprise value of ABC Company given the following model parameters...

Table 1: Model Parameters

Description	Value
Annualized revenue at month zero	10,000,0000
Assets as a percent of annualized revenue	60.00%
Annualized return on assets	15.00%
Annualized revenue growth rate	4.00%
Annualized cost of capital	12.00%

Question: What is the enterprise value of ABC Company?

Building Our Model

We will define free cash flow as follows...

$$\text{Free cash flow} = \text{New income} - \text{Reinvestment} \tag{1}$$

We will define the variable π to be the monthly return on assets. Using the parameters in Table 1 above the equation for monthly return on assets is...

$$\pi = \left(1 + \text{annualized return on assets}\right)^{\frac{1}{12}} - 1 = 1.15^{\frac{1}{12}} - 1 = 0.01171 \tag{2}$$

We will define the variable g to be the monthly revenue growth rate. Using Table 1 above the equation for the monthly revenue growth rate is...

$$g = \left(1 + \text{annualized revenue growth rate}\right)^{\frac{1}{12}} - 1 = 1.04^{\frac{1}{12}} - 1 = 0.00327 \tag{3}$$

We will define the variable k to be the monthly cost of capital. Using Table 1 above the equation for the monthly cost of capital is...

$$k = \left(1 + \text{annualized cost of capital}\right)^{\frac{1}{12}} - 1 = 1.12^{\frac{1}{12}} - 1 = 0.00949 \tag{4}$$

We will define the variable ϕ to be the ratio of assets to annualized revenue. Using Table 1 above the equation for the ratio of assets to annualized revenue is...

$$\phi = 0.60 \tag{5}$$

We will define the variable A_m to be assets at the end of month m and the variable R_m to be annualized revenue at the end of month m. Using Equations (3) and (5) above the equation for assets is...

$$A_m = \phi\, R_m = \phi\, R_0 \left(1+g\right)^m \tag{6}$$

We will define the variable N_m to be net income in month m. Using Equations (1), (2) and (6) above the equation for net income is...

$$N_m = \text{Monthly return on assets} \times \text{Assets at the beginning of month } m = \pi\, A_{m-1} = \pi\, \phi\, R_0 \left(1+g\right)^{m-1} \tag{7}$$

We will define the variable X_m to be reinvestment at the end of month m (i.e. earnings reinvested in the company so as to support a growing balance sheet). Using Equation (6) above the equation for reinvestment is...

$$X_m = \phi\, R_0 \left(1+g\right)^m - \phi\, R_0 \left(1+g\right)^{m-1} = \phi\, R_0 \left(1+g\right)^{m-1} \left((1+g)-1\right) = g\, \phi\, R_0 \left(1+g\right)^{m-1} \tag{8}$$

We will define the variable C_m to be free cash flow in month m that is available for distribution at the end of month m. Using Equations (1), (7) and (8) above the revised equation for free cash flow is...

$$C_m = \pi\, \phi\, R_0 \left(1+g\right)^{m-1} - g\, \phi\, R_0 \left(1+g\right)^{m-1} = \phi\, R_0 \left(1+g\right)^{m-1} \left(\pi - g\right) \tag{9}$$

We will define the variable V_0 to be enterprise value at time zero. Using Equations (4) and (9) above the equation for enterprise value is...

$$V_0 = \sum_{m=1}^{\infty} C_m \left(1+k\right)^{-m} = \sum_{m=1}^{\infty} \phi\, R_0 \left(\pi - g\right) \left(1+g\right)^{m-1} \left(1+k\right)^{-m} \tag{10}$$

Note that we can rewrite Equation (10) above as...

$$V_0 = \phi\, R_0\, \frac{\pi - g}{1+g} \sum_{m=1}^{\infty} \theta^m \quad \text{...where...} \quad \theta = \frac{1+g}{1+k} \tag{11}$$

The solution to a polylogarithm of order zero over the interval $[m=1, m=\infty]$ is... [1]

$$\sum_{m=1}^{\infty} \theta^m = \frac{\theta}{1-\theta} = \frac{\frac{1+g}{1+k}}{1-\frac{1+g}{1+k}} = \frac{1+g}{k-g} \quad \text{...where...} \quad 0 < \theta < 1 \tag{12}$$

Using Equation (12) above we can rewrite enterprise value Equation (11) above as...

$$V_0 = \phi\, R_0\, \frac{\pi - g}{1+g}\, \frac{1+g}{k-g} = \phi\, R_0\, \frac{\pi - g}{k-g} \tag{13}$$

The Answer To Our Hypothetical Problem

Using Equation (13) above and the parameter estimates above the answer to our hypothetical problem is...

$$V_0 = \phi\, R_0\, \frac{\pi - g}{k-g} = 0.60 \times 10,000,000 \times \frac{0.01171 - 0.00327}{0.00949 - 0.00327} = 8,149,000 \tag{14}$$

References

[1] Gary Schurman, *Polylogarithms of Order Zero,* May, 2019